Basic Vehicle Performance Modeling

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Overview

• Basic 2 axle vehicle model
• Review typical road loads
• Example modeling for performance
Performance Modeling

• Performance usually relates to longitudinal motion of a vehicle, and is constrained by one of two limits.
  - The first is a power plant limitation, which tends to be especially critical at high speed.
  - The second limitation is traction, which tends to dominate at low speeds.
• In both cases, the impact on acceleration or deceleration is of particular interest.
• Some common needs might be:
  - Finding torque and power for a given application (e.g., to go up a hill, drawbar)
  - Selecting a power plant
  - Development or evaluation of cruise, braking, traction, or engine control
Basic Model – 2 axle vehicle

Along the longitudinal (x) axis:

\[ \dot{p}_x = m \frac{dv_x}{dt} = m \cdot a_x = \sum F_x \]

\[ \sum F_x = \text{Tractive force} - \text{Road Loads} \]

\[ \sum F_x = F_f + F_r - R_a - R_{rf} - R_{rr} - R_d - R_g \]

- \( F_{f,r} \) = tractive effort on front and rear
- \( R_a \) = aerodynamic resistance force
- \( R_{rf,rr} \) = rolling resistance on front and rear
- \( R_d \) = drawbar load
- \( R_g \) = grade resistance = \( W \sin \theta_s \)

From Wong, Chapter 3, Fig. 3.1
Basic Model – 2 axle vehicle

Use equilibrium conditions in the vertical direction and about the \( y \) axis.

\[
\dot{p}_z = m \frac{dv_z}{dt} = 0 = \sum F_z
\]

\[
\dot{h}_y = I_y \frac{d\omega_y}{dt} = 0 = \sum T_y
\]

Need to:
1. Find loads on the axles
2. Use knowledge of road adhesion and other vehicle parameters to determine tractive effort (TE)
Dissipative Loads

- Recall, any **dissipative** forces take the form of **effort** vs. **flow**.
- For **translation**: force $(F)$ – velocity $(V)$ plots
- For **rotation**: torque $(T)$ – (angular) speed $(\omega)$ plots
Aerodynamic Forces and Moments

Aerodynamic effects can affect vehicular dynamics in several ways.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Force</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal, positive rearward</td>
<td>Drag force</td>
<td>Rolling moment</td>
</tr>
<tr>
<td>Lateral, positive right</td>
<td>Side force</td>
<td>Pitching moment</td>
</tr>
<tr>
<td>Vertical, positive upward</td>
<td>Lift force</td>
<td>Yawing moment</td>
</tr>
</tbody>
</table>

Refer to Wong, Section 3.2 or Gillespie, Ch. 4, for a detailed discussion of aerodynamic effects.
Tire Force and Moment Conventions

Gillespie (1992)

Fig. 10.3 SAE tire axis system.

Wong (1993/2001)
Rolling Resistance

- Primarily caused by hysteresis in tire materials due to deflection of the carcass while rolling.
- Others factors that might contribute to RR in tires: friction in sliding, air circulation, fan effect of rolling tire.
- Dominant load at low speeds; dependent on speed.
- Example: loads on tire at 80-95 mph (90-95% hysteresis, 2-10% friction, 1.5-3.5% air resistance)
- For free-rolling tire, a horizontal force is introduced to balance the ‘rolling resistance moment’, which arises when pressure shifts to leading half due to carcass deflection.
- Ratio of rolling resistance to normal load is the coefficient of rolling resistance, which incorporates all the complicated and interdependent physical properties of tire and ground.
- Aerodynamics become equal to rolling at about 50 or 60 mph.
Typical Road Loads
From Steeds, “Mechanics of Road Vehicles” (1960)

Generally, we assume that aerodynamic effects can be ignored for low ground speeds.

Rolling resistance and grade dominate until about 50 to 60 mph.
Estimating Rolling Resistance

Ex. Radial-ply passenger car tires under rated loads and inflation pressures on a smooth road (Wong, Ch.1), \[ f_r = 0.0136 + 0.4 \times 10^{-7} V^2 \]
with \( V \) in km/h.

Total force is estimated on all wheels, \[ F_r = F_{rf} + F_{rr} = f_r W \]
where \( W \) is the total weight on the vehicle.

In some cases, can approximate with average values, as given the table shown.

<table>
<thead>
<tr>
<th>Road surface</th>
<th>Coefficient of rolling resistance ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pneumatic car tires on</td>
<td></td>
</tr>
<tr>
<td>Large sett pavement</td>
<td>0.015</td>
</tr>
<tr>
<td>Small sett pavement</td>
<td>0.015</td>
</tr>
<tr>
<td>Concrete, asphalt</td>
<td>0.013</td>
</tr>
<tr>
<td>Rolled gravel</td>
<td>0.02</td>
</tr>
<tr>
<td>Tarmacadam</td>
<td>0.025</td>
</tr>
<tr>
<td>Unpaved road</td>
<td>0.05</td>
</tr>
<tr>
<td>Field</td>
<td>0.1...0.35</td>
</tr>
<tr>
<td>Pneumatic truck tires on</td>
<td></td>
</tr>
<tr>
<td>concrete, asphalt</td>
<td>0.006...0.01</td>
</tr>
<tr>
<td>Strake wheels in field</td>
<td>0.14...0.24</td>
</tr>
<tr>
<td>Track-type tractor in field</td>
<td>0.07...0.12</td>
</tr>
<tr>
<td>Wheel on rail</td>
<td>0.001...0.002</td>
</tr>
</tbody>
</table>

From Automotive Handbook (SAE)
Additional Estimates

Speed dependence of rolling resistance

For low speeds: \( f_r = 0.01(1 + V / 100) \), \((V \text{ in mph})\)

At higher speeds: \( f_r = f_o + 3.24 f_s \left( \frac{V}{100} \right)^{2.5} \), \((V \text{ in mph})\)

\( f_o \) = basic coefficient

\( f_s \) = speed coefficient

From Automotive Handbook (SAE)
3.1 A vehicle weighs 20.02 kN (4500 lb) and has a wheelbase of 279.4 cm (110 in.). The center of gravity is 127 cm (50 in.) behind the front axle and 50.8 cm (20 in.) above ground level. The frontal area of the vehicle is 2.32 m² (25 ft²) and the aerodynamic drag coefficient is 0.45. The coefficient of rolling resistance is given by \( f_r = 0.0136 + 0.4 \times 10^{-7} V^2 \), where \( V \) is the speed of the vehicle in kilometers per hour. The rolling radius of the tires is 33 cm (13 in.). The coefficient of road adhesion is 0.8. Estimate the possible maximum speed of the vehicle on level ground and on a grade of 25% as determined by the maximum tractive effort that the tire-road contact can support if the vehicle is (a) rear-wheel-drive, and (b) front-wheel-drive. Plot the resultant resistance versus vehicle speed, and show the maximum thrust of the vehicle with the two types of drive.

From Wong (2001)
Vehicle on an Incline – 2

These equations can be formulated to solve for the 3 unknowns:

\[
\begin{pmatrix}
W_f \\
W_r \\
a_x
\end{pmatrix} = \begin{pmatrix}
-h \cdot F_t + \frac{(b + h \cdot f_r)}{L} \cdot W \cdot \cos(\theta) \\
h \cdot F_t + \frac{(a - h \cdot f_r)}{L} \cdot W \cdot \cos(\theta) \\
-\frac{f_r}{m} \cdot W \cdot \cos(\theta) - \frac{1}{m} \cdot (W \cdot \sin(\theta) + F_a - F_r)
\end{pmatrix}
\]
Vehicle on an Incline – 3

Data for Problem 3.1 from Wong (1993)

\[ W := 4500 \text{lbf} \quad L := 279.4 \text{cm} \quad A_f := 2.32 \text{m}^2 \]
\[ m_v := \frac{W}{g} \quad h := 50.8 \text{cm} \quad C_d := 0.45 \]
\[ \mu := 0.8 \quad a := 127 \text{cm} \quad \rho := 1.23 \frac{\text{kg}}{\text{m}^3} \]
\[ b := L - a \quad B := 0.4 \times 10^{-7} \cdot \frac{1}{\left(\frac{\text{km}}{\text{hr}}\right)^2} \]
\[ r := 33 \text{cm} \]

\[ F_g(\theta) := W \cdot \sin(\theta) \]
\[ f_f(v) := A + B \cdot v^2 \]
\[ F_a(v) := \frac{1}{2} \cdot \rho \cdot C_d \cdot A_f \cdot (v)^2 \]
\[ \theta := \arctan\left(\frac{1}{4}\right) \]
\[ F_{\text{max}}(v) := \mu \cdot W \cdot \cos(\theta) \cdot \frac{(a - h \cdot f_f(v))}{(L - h \cdot \mu)} \]
\[ F_{\text{max}}(v) := \mu \cdot W \cdot \cos(\theta) \cdot \frac{(b + h \cdot f_f(v))}{(L + h \cdot \mu)} \]
\[ \theta = 14.036 \text{deg} \]
Vehicle on an Incline – 4

Graphically extracted results

<table>
<thead>
<tr>
<th>Pt.</th>
<th>F_{\text{max}} (N)</th>
<th>V (m/s)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7493</td>
<td>60</td>
<td>FWD</td>
</tr>
<tr>
<td>2</td>
<td>8274</td>
<td>69</td>
<td>RWD</td>
</tr>
<tr>
<td>3</td>
<td>7484</td>
<td>105</td>
<td>FWD</td>
</tr>
<tr>
<td>4</td>
<td>8267</td>
<td>111</td>
<td>RWD</td>
</tr>
</tbody>
</table>

Check out the power delivery.

Line 2 = 571 kW! (766 hp)

Does this seem reasonable?
Summary – Road Loads

• This overview of road loads illustrates the large amount of information available related to predicting and controlling typical road loads.

• Basic analysis can be conducted to determine acceleration performance on grades under aerodynamic loading, including the effect of rolling resistance.

• Later, we will examine how power plant and transmission effects play a role in longitudinal performance.

• First, we will study tire-ground interaction in more detail to understand braking and traction applications.
References


